

On magnetization of quark-gluon plasma at the LHC experiment energies

V. Skalozub*

Oles Honchar Dnipropetrovsk National University, 49010 Dnipro, Ukraine

P. Minaiev†

Oles Honchar Dnipropetrovsk National University, 49010 Dnipro, Ukraine

Abstract

Large scale chromomagnetic, B_3, B_8 , and usual magnetic, H , fields have to be generated in QCD after the deconfinement phase transition (DPT) at temperatures T larger than deconfinement temperature T_d . The two former fields are created spontaneously due to asymptotic freedom of gluon interactions. Whereas H is produced due to either the feature of quarks to possess both electric and color charges or a vacuum polarization in this case. At the polarization, the vacuum quark loops mix the external fields. As a result, B_3, B_8 become the sources generating H . The latter field appears at T much lower than the electroweak phase transition temperature T_{ew} . This mechanism should exhibit itself at the LHC experiments on heavy ion collisions. It operates at the one-loop diagram level for an effective potential. The created fields are temperature dependent and occupying the macroscopic volume of quark-gluon plasma. The magnetization influences different processes and may serve as a signal for the DPT .

1 Introduction

Studying of QCD vacuum, its dynamics and properties, is one of the most important problems investigated at the LHC experiments. Here, in heavy ion collisions a new matter phase - quark-gluon plasma (QGP)- is expected to be produced. It consists of quarks and gluons deliberated from hadrons at high temperature $T > T_d \sim 180 - 200$ MeV, where T_d is a deconfinement temperature. In theory, investigation of this phase transition and QGP properties was carried out by different method - analytic perturbative and nonperturbative, various numerical methods and Monte-Carlo simulations on a lattice. Details on these studies and results have been presented in numerous publications (see, for example [1] - [6]). In particular, QGP had been existed in the hot Universe and influenced various processes [8], [9]. One of the distinguishable properties of gluon fields at high temperature is a spontaneous vacuum magnetization closely related with asymptotic freedom. It happen due to a large magnetic moment of charged color gluons (gyromagnetic ratio $\gamma = 2$) and results in a stable, temperature dependent classical chromo(magnetic) fields occupying large domains of space. The magnetization has been investigated in detail in $SU(3)$ gluodynamics [10] by analytic methods and in $SU(2)$ gluodynamics [11], [13] by the Monte-Carlo simulations on a lattice. In both cases, the creation of the fields was detected.

In the present paper, we investigate a scenario corresponding to experiments on heavy ion collisions at the LHC. It is realized in QGP at temperatures $T > T_d$ and related with quarks. The effect consists in generation of usual magnetic field H due to the vacuum polarization of quark fields by the constant color magnetic fields B_3 and B_8 , which have to be created spontaneously in the gluon sector after the DPT . Really, quarks possess both electric e and color g charges. Therefore, due to vacuum polarization, the mixing of external fields happens and the corresponding terms in an effective potential may serve as specific sources for H . This mechanism has to start to operate at temperatures of QGP $T_d < T \ll T_{ew}$, where $T_{ew} \sim 100$ GeV is an electroweak phase transition temperature. At these T the non-Abelian $SU(2)$ constituent of the electromagnetic field is screened by the scalar condensate. So, the field H could not be generated spontaneously in the electroweak sector of the standard model [15]. It is expected that the magnetic field H is temperature dependent and occupying a large plasma volume as

*e-mail: Skalozubv@daad-alumni.de

†e-mail: Minaevp9595@gmail.com

B_3 and B_8 do. According to this scenario, the color magnetic fields, generated spontaneously after the DPT , play the role of the specific sources for usual magnetic field.

To investigate this possibility, we calculate the one-loop contribution to the effective potential $V^{(1)}(B_3, B_8, H, T)$ of quark fields in the external color magnetic fields B_3 and B_8 and usual magnetic field H at finite temperature and investigate its properties for $T > T_d$. Two specific approximations will be applied. First is high temperature expansion for the effective potential derived by means of the Mellin transformation. Second is a low temperature expansion which also is relevant. In this way the effective potentials of the external fields generated by the quark loop are derived. Assuming that the fields $B_3(T)$ and $B_8(T)$ appear spontaneously after the DPT due to vacuum polarization of gluon fields, as it was derived in [10], and taking into consideration the specific values of these fields corresponding to temperatures not much higher of T_d , we estimate the strength of the magnetic field H .

The paper is organized as follows. In sect. 2 the one-loop effective potential $V^{(1)}(B_3, B_8, H, T)$ of external fields and temperature generated by quark contributions is calculated. In sect. 3 the high temperature and low temperature asymptotic expansions are derived. Then, in sect. 4 the generated magnetic field H is investigated. Discussion and conclusions are given in the last section.

2 Effective potential of external fields

In what follows, we consider the situation when the temperature of the QGP is not much higher than T_d . So, according to [10], both color magnetic fields B_3 and B_8 are spontaneously created in the gluon sector of QCD . They will be considered as the background for calculating the one-loop quark effective potential $V^{(1)}(B_3, B_8, H, T)$.

To be in correspondence with the notation and results of this paper, we present the $SU(3)_c$ gluon field in the form

$$A_\mu^a = B_\mu^a + Q_\mu^a, \quad (1)$$

where B_μ^a is background classical field and Q_μ^a presents quantum gluons. We choose the external field potential in the form $B_\mu^a = \delta^{a3}B_{3\mu} + \delta^{a8}B_{8\mu}$, where $B_{3\mu} = H_3\delta_{\mu 2}x_1$ and $B_{8\mu} = H_8\delta_{\mu 2}x_1$ describe constant chromomagnetic fields directed along third axis in the Euclidean space and $a = 3$ and $a = 8$ in the color $SU(3)_c$ space, respectively. The field tensor has components: $F_{\mu\nu}^{ext\ a} = \delta^{a3}F_{3\mu\nu} + \delta^{a8}F_{8\mu\nu}$, $F_{c12} = -F_{c21} = H_c$, $c = 3, 8$. Accounting for this choice, we direct usual magnetic field also along third axis and write the electromagnetic potential in the similar form: $A_\mu^{ext} = H\delta_{\mu 2}x_1$.

In [10] it was obtained that at temperatures $T \geq T_d$ both the fields

$$\begin{aligned} gH_3(T) &= 0.2976 \frac{g^4 T^2}{\pi^2}, \\ gH_8(T) &= 0.9989 \frac{g^4 T^2}{\pi^2} \end{aligned} \quad (2)$$

are spontaneously generated. This situation is changed at asymptotically high temperatures when only the field H_3 is created. Below, we consider the first more general case and concentrate on the qualitative picture of the phenomenon. We take into consideration these fields as external ones without accounting of the quark corrections to the values (2). They are numerically small and do not change significantly final results.

To consider the generation of H by the fields H_3, H_8 we first calculate the quark spectrum in the presence of all these fields. The corresponding Dirac equation reads

$$(i\gamma_\mu D_\mu + m_f)\psi^a = 0, \quad (3)$$

where ψ^a is a quark wave function, a is color index, m_f is mass of f -flavor quark. The covariant derivative describes the interactions with external magnetic fields H and H_3, H_8 :

$$D_\mu = \partial_\mu - ieA_\mu^{ext} - ig(T^3 B_\mu^3 + T^8 B_\mu^8) \quad (4)$$

where $T^3 = \frac{\lambda^3}{2}$, $T^8 = \frac{\lambda^8}{2}$ are the generators of $SU(3)$ group, $\lambda^{3,8}$ are Gell-Mann matrixes. Due to the choice of the potentials and the Abelian nature of the fields we can present the quark spectrum as the sum of contributions of the three following external fields:

$$\begin{aligned} qH_1 &= q_f H + \frac{g}{2}(H_3 + \frac{1}{\sqrt{3}}H_8), \\ qH_2 &= q_f H + \frac{g}{2}(-H_3 + \frac{1}{\sqrt{3}}H_8), \end{aligned} \quad (5)$$

$$qH_3 = q_f H - g \frac{1}{\sqrt{3}} H_8.$$

Here, q_f is electric charge of f -quark. Each flavor energy spectrum is given by the known expression (see, for instant, [12])

$$\epsilon_{i,n,\rho,f}^2 = m_f^2 + p_z^2 + (2n+1)qH_i - \rho qH_i, \quad (6)$$

where p_z is momentum along the field direction, $\rho = \pm 1$.

Vacuum energy is defined as the sum of the modes having negative energy, which also is well known [12]

$$V_{l=0}^{(0)} = \frac{1}{8\pi^2} \sum_{f=1}^6 \sum_{i=1}^3 \int_0^\infty \frac{ds}{s^3} e^{-m_f^2 s} [qH_i s \coth(qH_i s) - 1]. \quad (7)$$

Next step is to account for finite temperature. In the imaginary time formalism for fermions, it is reduced to the summation over discrete odd imaginary energy $p_4 = \frac{(2l+1)\pi}{\beta}$, $\beta = 1/T$ is inverse temperature [3], [1]. The result yields

$$V = \frac{1}{8\pi^2} \sum_{f=1}^6 \sum_{i=1}^3 \sum_{l=-\infty}^\infty (-1)^l \int_0^\infty \frac{ds}{s^3} \exp(-m_f^2 s - \frac{\beta^2 l^2}{4s}) [qH_i s \coth(qH_i s) - 1]. \quad (8)$$

This is expression of interest. It will be investigated in the next section. Note, the term with $l = 0$ is the vacuum energy (7).

3 Asymptotic expansion

The expression (8) includes complete information about the polarization. In particular, it describes the nonlinear effective potentials of field interactions. To derive them at finite temperature we make a high temperature and low temperature expansion of (8). In the first case, we apply Millin's transformation for summing up the series in (8). This method is described in detail in [14]. In what follows, we use the dimensionless variables and measure all the parameters in units of proton mass m_p . The dimensionless constituent quark mass is $m_f = m_f/m_p$, the field strengths $h_i = (qH_i)/m_p^2$, and the temperature $T = T/m_p$. In these units the dimensionless potential (8) is V/m_p^4 .

Expanding $\coth(qH_i s)$ in series, performing integration over s and applying Mellin's transformation for the obtained expressions we find for the temperature dependent part in the high temperature limit ($T \rightarrow \infty$)

$$\begin{aligned} 12\pi^2 V^{(T)} &= \frac{2}{3} \sum_i h_i^2 \left[\frac{1}{2} \left(\gamma + \ln\left(\frac{\omega}{\pi}\right) \right) + \frac{7\zeta'(-2)}{4} \omega^2 + \frac{31\zeta'(-4)}{64} \omega^4 \right] \\ &- \frac{1}{90\mu^4} \sum_i h_i^4 \left[-1 + \frac{31\zeta'(-4)}{8} \omega^4 \right]. \end{aligned} \quad (9)$$

In the low temperature limit ($T \rightarrow 0$) we obtain

$$\begin{aligned} 12\pi^2 V^{(T)} &= -\frac{2}{3} \sum_i h_i^2 \sqrt{\frac{\pi}{2}} e^{-\omega} \left[\frac{1}{\sqrt{\omega}} - \frac{1}{8\omega\sqrt{\omega}} \right] \\ &+ \frac{1}{90\mu^4} \sum_i h_i^4 \sqrt{\frac{\pi}{2}} e^{-\omega} \left[\omega\sqrt{\omega} + \frac{15}{8}\sqrt{\omega} + \sqrt{\frac{1}{\omega} \frac{105}{128}} \right], \end{aligned} \quad (10)$$

where h_i are dimensionless fields (5). We also introduced the notations:

$$x_3 = \frac{B_3 Q_c}{m_p^2}, \quad x_8 = \frac{B_8 Q_c}{m_p^2}, \quad x = \frac{He}{m_p^2}, \quad (11)$$

$$\omega = \beta m_f = \mu m_p \beta, \quad \mu = \frac{m_f}{m_p}. \quad (12)$$

From (9), (10) we see that different kind mixing of fields takes place. In particular, there are the terms of the form

$$\sim eH \times (gH_3)^3, \sim eH \times (gH_8)^3, \dots, \quad (13)$$

which mean that color fields H_3, H_8 play the role of external sources (magnetic moments, etc.) for usual magnetic field H and generate it. To verify this idea, we have to consider the tree-level plus V effective potentials $V_{eff.} = \frac{1}{2}H^2 + V(H, H_3, H_8, T)$ and solve the stationary equation

$$\frac{\partial V_{eff.}}{\partial H} = 0. \quad (14)$$

If it has the nonzero solutions $H = H_c$ and the energy of them is negative, we determine the magnetic field produced after the DPT. Investigation of this will be done in the next section.

4 Generation of magnetic field

Let us present the solutions to equation (14) in high temperature (9) and low temperature (10) approximations to the effective potential. From the point of view of applications, the former case is more adequate to QGP at the LHC experiments.

In Table 1 we show the values of H_3, H_8 obtained according to formulae (2) and H generated according to (14) at temperatures $T = 180, 200, 220$ MeV corresponding to $T \geq T_d$. The case of only one constituent quark mass m_u is investigated. The values of the magnetic field energy are also adduced. In units of m_p and assuming for estimates that $\alpha_s = \frac{g^2}{4\pi} = 1$ the color magnetic field strengths read

$$x_3 = \frac{gH_3}{m_p^2} = 0.76164 \frac{T^2}{m_p^2}, \quad x_8 = \frac{gH_8}{m_p^2} = 15.9824 \frac{T^2}{m_p^2}. \quad (15)$$

T, MeV	x_8	x_3	$V_{eff}^{(h)} 10^{-2}$	$x^{(h)} 10^{-3}$	$V_{eff}^{(l)} 10^{-2}$	$x^l 10^{-3}$
180	0.59	0.18	-0.153	-0.546	-0.150	-0.531
200	0.73	0.22	-0.211	-0.575	-0.202	-0.546
220	0.88	0.26	-0.272	-0.582	-0.251	-0.528

Table 1. The strength values of the fields generated at the typical temperatures

The solutions for considered asymptotic expressions are close to each other. So, both these expansions are equally applicable. The energy of magnetic field is negative decreasing function of T . The total energy of fields H_3, H_8 and H is also negative. So, these field configurations have to be energetically favorable. As we see, the field strength values h are two order smaller and orientation is opposite to the direction of the color field ones.

As a result, all this is signaling the generation of macroscopic magnetic field H in the presence of color magnetic fields H_3, H_8 , which act as the sources for it. The field is occupying all the QGP volume where the color fields present. Of course, these are estimates. More detailed analysis has to account for all the quark flavors. But in this paper we concentrate on the qualitative picture of the phenomenon. We expect that only quantitative changes could happen.

5 Discussion and conclusion

As it follows from the obtained results, at temperatures $T > T_d$ in QGP either large scale Abelian chromomagnetic H_3, H_8 or usual magnetic H fields have to present. The mechanisms for generation of them are quite different. The color fields are generated at high temperature in the $SU(3)_c$ sector of the standard model due to gluon vacuum polarization [10], [13]. They exist in space till the color is screened at low temperature. Formally, such type fields are solutions to field equations without sources. The $SU(2)$ component of usual magnetic field is also produced spontaneously at temperatures T larger than electroweak phase transition temperature $T_{ew} \sim 100$ GeV due to W -boson vacuum polarization. But it is screened by the scalar field condensate appeared after this phase transition [15].

With temperature lowering, in the interval $T_{ew} > T > T_d$, when color magnetic fields are present, the field H esquires other mechanism for generation. It is induced by the vacuum polarization of quark fields, as it was shown above. Due to nonlinearity of the polarization, the color fields H_3, H_8 become the sources producing H . All these fields occupy macroscopic volumes of QGP . One of consequences of the magnetization is the discrete spectra of color and/or electric charged particles, as it is shown for quarks in (6). In fact, this is the distinguishable feature

of *QGP*. At low temperatures, after the confinement the macroscopic magnetic fields are screened. Only the fields produced by charged currents remain.

As corollary, we have derived that at the *LHC* experiment energies the *QGP* has to be magnetized, that may serve as signal for the *DPT*. To give concrete numerical estimates one has to account for other quark flavors and long range correlation corrections. All these contributions to the effective potential as well as specific processes signaling the plasma magnetization will be reported elsewhere.

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